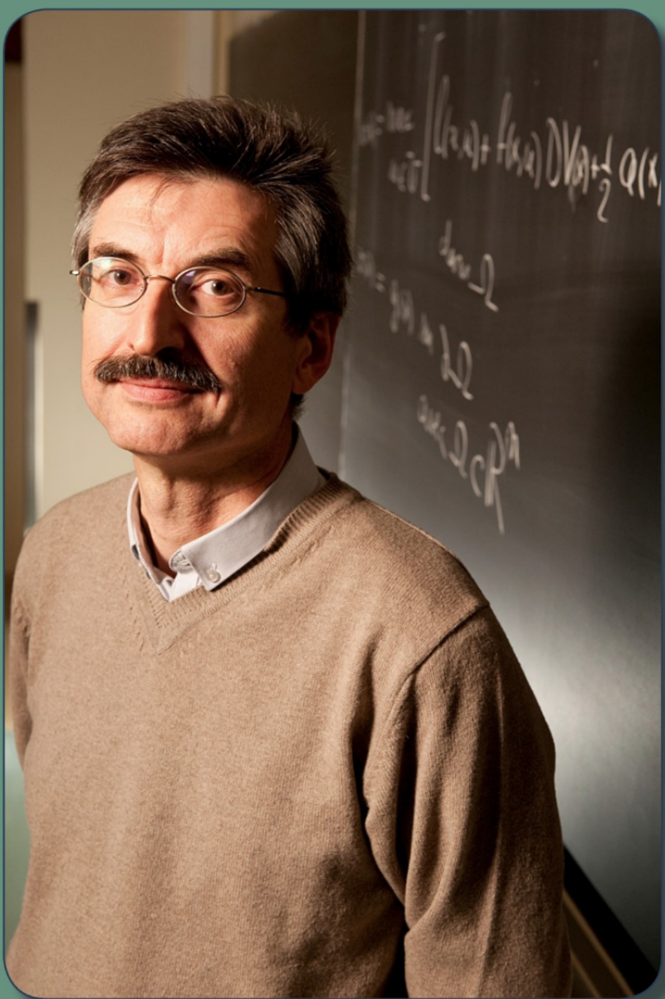
CONTROL AND OPTIMIZATION CONFERENCE
On the occasion of Frédéric Bonnans 60th birthday

15-17 November 2017 - EDF Laboratories, Palaiseau



CONTROL AND OPTIMIZATION

The aim of this event is to present some recent developments in the field of Optimal Control and Optimization, including numerical and theoretical results, as well as applications to finance, electricity production and management, among others.

The list of topics will include:

- Optimal Control
- Optimization and Sensitivity Analysis
- Numerical methods for Optimization and Optimal Control
- Stochastic Programming and Control
- Industrial Applications

ADDRESS
EDF Lab Paris-Saclay
7 Boulevard Gaspard Monge
91120 Palaiseau
France

INFORMATION
Check our website: bonnans60.sciencesconf.org
Contact: bonnans60@sciencesconf.org

CMAP
EDF
inria
inventors for the digital world
ENSTA ParisTech
Programme
PGMO
Pour l'optimisation et la recherche opérationnelle

Control and Optimization Conference on the occasion of Frédéric Bonnans 60th birthday

Novembre 15 **- November** 17**,** 2017
Palaiseau, France

BOOK OF ABSTRACTS

Contents

Introduction

Welcome to the Control and Optimization Conference on the occasion of Frédéric Bonnans 60th birthday. Frédéric is a leading expert in Optimization and Optimal Control theories and the aim of this conference is to celebrate Frédéric's scientific achievements by bringing together many of his collaborators and friends.

The topics covered in this conference are:

- Optimal Control.
- Optimization and Sensitivity Analysis.
- Numerical methods for Optimization and Optimal Control.
- Stochastic Programming and Control.
- Industrial Applications.

It is a great honour for us to organize this three-days event and we expect that it will also be a fruitful and pleasant scientific experience for you.

Joyeux anniversaire Frédéric!

The organizing committee,

María Soledad Aronna, Héctor Ramírez, Francisco Silva and Hasnaa Zidani.

Organizing Committee

Workshop Organizers

María Soledad Aronna	Escola de Matemática Aplicada, EMAp/FGV, Brazil.
Héctor Ramírez	Mathematical Engineering Department and Center for Mathematical Modeling, Universidad de Chile, Chile.
Francisco Silva	DMI-XLIM, Université de Limoges and TSE-R, Université de Toulouse I, France.
Hasnaa Zidani	ENSTA ParisTech, France.

Administrative management

Jessica Gameiro	INRIA Saclay, France.

Practical information

- The conference will take place in the **lecture hall 1 (amphithéâtre 1)** of the building **AZUR** at **EDF Labs**.
- The coffee breaks, lunch and the cocktail will take place in the **"LE FOYER DE L'AUDITORIUM"** in the first floor of the building AZUR.
- Here is a plan of the campus.

- Further informations on the AZUR building.

AZUR

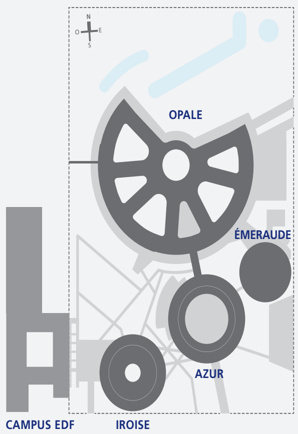
In addition to being a visitor reception building, Azur is also a conference centre, hosting on its ground floor an auditorium, two amphitheatres, a vast reception area, a business centre and the concierge services. On its first floor, the Azur building houses meeting and training rooms.

IROISE

Also located in the open area, this building is dedicated to food services (company canteen and brewery).

ÉMERAUDE

Located in the access restricted area, the Emeraude building houses the experimental and testing facilities. It also houses laboratories, offices, archives and technical rooms.



1. EDF Lab Paris-Saclay
2. Poste source ERDF (2017)
3. DigitéoLabs
4. AgroParisTech INRA
5. Institut Mines-Télécom (2019)
6. Nano - INNOV System X
7. Centre de Nanosciences et Nanotechnologies (2017)
8. Horiba Jobin-Yvon
9. Institut Photovoltaïque d'Île-de-France (2017)
10. Ecole Polytechnique
11. Bâtiment d'enseignements mutualisés (2018)
12. ENSTA ParisTech (2016)
13. Danone Research
14. ENSTA ParisTech
15. Thales
16. Pôle de mécanique (2019)

- Some indications on how to arrive to the campus.

ACCESS EDF LAB PARIS-SACLAY:

Open from 7 AM to 8 PM (from Monday to Friday)

GPS: 48.717686, 2.198853

- Pedestrian access: 7 Boulevard Gaspard Monge, Palaiseau
- Access to the staff parking and the delivery area: 2 and respectively 4 rue Rosalind Franklin in Palaiseau
- Visitors parking: in front of the main entrance, 7 Boulevard Gaspard Monge in Palaiseau

ACCESS BY PUBLIC TRANSPORTATION

From Paris

RER B or C > Massy Palaiseau stop;

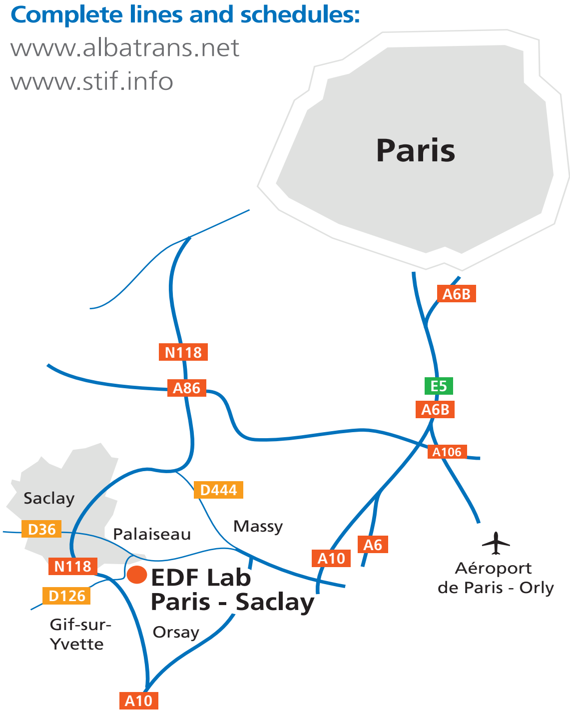
Then **Bus 91.06 (or 91.10)** > Palaiseau-Campus stop

From Vélizy II – Gare routière T6 station

Take the Express 91.08

Complete lines and schedules:

www.albatrans.net
www.stif.info



ACCESS BY CAR

From Paris

→ Take the N118 from Pont de Sèvres.
Take the 9th exit, towards "Orsay-le-Guichet".

→ From Porte d'Orléans:
take the A6 highway towards Lyon.
Continue on the A10 highway towards "Nantes/Bordeaux".
Take the "D444/Versailles/ Igny/Bièvres" exit on the left. Continue on the D36.

From Paris airports

→ From Paris-Charles de Gaulle:
take the A1 highway towards Paris.
At Porte de la Chapelle, take the exterior ring road towards Rouen.
At Porte de Saint Cloud, take the N10 towards "Bordeaux/Nantes". Then follow instructions as for the N118 car access.

→ From Paris-Orly:
take the N7 towards Paris. Then follow the A86 highway towards Versailles.
Take the 30th exit towards « A6/A10/Bièvres/ Igny/Bordeaux/Nantes ».
Continue on the A10 highway towards « Nantes/Bordeaux ».
Take the "D444/Versailles/ Igny/Bièvres" exit on the left. Continue on the D36.

EDF Direction Recherche et Développement
EDF Lab Paris-Saclay

7 boulevard Gaspard Monge
91120 Palaiseau
Tél : 01 78 19 32 00

Program at a glance

	Wednesday 15th	*Thursday 16th*	*Friday 17th*
09:15 - 09.50	Registration	R. Vinter	A. Ruszczynski
09:50 - 10:25	Opening	N. P. Osmolovskii	R. Henrion
10:25 - 11:00	R. Cominetti	A. Dmitruk	W. van Ackooij
11:05 - 11:35	***Coffee Break***	***Coffee Break***	***Coffee Break***
11:35 - 12:10	G. Allaire	E. Zuazua	A. Sulem
12:10 - 12:45	P. Rouchon	J.P. Raymond	N. Touzi
12:45 - 14:15	***Lunch***	***Lunch***	***Lunch***
14:15 - 14:50	E. Casas	P. Martinon	
14:50 - 15:25	F. Tröltzsch	M. Haddou	
15:25 -15:40	***A short Break***	***A short Break***	
15:40 - 16:15	D. Tiba	M. Mnif	
16:15 - 16:50	E. Trélat	T. Bayen	
16:50 - 17h25		S. Gaubert	
17:30 - 19:00		Cocktail reception at EDF-Labs	

Abstracts

Wednesday, 15 November

Rates of convergence for the Krasnoselskii-Mann fixed point iteration

Roberto Cominetti
University Adolfo Ibañez
roberto.cominetti@uai.cl

Joint work with joint work M. Bravo, M. Pavez-Signé, J. Soto, and J. Vaisman.

We analyze the convergence of an inexact version of the classical Krasnoselskii-Mann iteration for computing fixed points of nonexpansive maps in Banach spaces. Our main result establishes a new metric bound for the fixed-point residuals, from which we derive their rate of convergence as well as the convergence of the iterates towards a fixed point. To this end we consider a nested family of optimal transport problems that provide a recursive bound for the distance between the iterates. These recursive bounds are in turn interpreted as expected rewards for an underlying Markov chain, which leads to explicit rates of convergence. In the case of the exact iteration we show that these bounds are tight by building a nonexpansive map that attains them with equality, and we deduce that the optimal constant of asymptotic regularity is exactly $1/\sqrt{\pi}$. The results are extended to continuous time to study the asymptotics of nonautonomous evolution equations governed by nonexpansive operators.

Shape optimization of a layer by layer mechanical constraint for additive manufacturing

Grégoire Allaire
CMAP, Ecole Polytechnique, F-91128 Palaiseau, France
gregoire.allaire@polytechnique.fr

Joint work with C. Dapogny, R. Estevez, A. Faure and G. Michailidis.

The purpose of this talk is to report on a recent work [1, 2], in collaboration with C. Dapogny, R. Estevez, A. Faure and G. Michailidis, which introduced a new functional of the domain, to be used in shape optimization problems as a means to enforce the constructibility of shapes by additive manufacturing processes. Additive manufacturing (or 3-d printing) is a common label for quite different methodologies, which share the fact that the construction process starts with a slicing procedure: the input shape is converted into a series of two-dimensional layers which are sequentially built. We focus on powder bed fusion methods used to process metals; at the beginning of the construction of each layer, metallic powder is spread within the build chamber and a laser (or an electron beam) is used to bind the grains together. Although this technique can build very complex structures, several problems remain and one of them is that it is impossible to build shapes showing large overhangs, i.e. regions hanging over void (or powder) without sufficient support from the lower structure. To avoid this undesirable effect we propose a constraint functional which aggregates the self-weights of all the intermediate structures of the shape appearing in the course of its layer by layer assembly. Its shape derivative is computed by the adjoint method and an algorithm is proposed to accelerate the significant computational effort entailed by the implementation of these ideas. Several 2-d and 3-d numerical examples of shape optimization are discussed.

References

[1] G. Allaire, C. Dapogny, A. Faure, G. Michailidis, *Shape optimization of a layer by layer mechanical constraint for additive manufacturing*, C. R. Math. Acad. Sci. Paris, 355, no. 6, 699-717 (2017).

[2] G. Allaire, C. Dapogny, R. Estevez, A. Faure and G. Michailidis, *Structural optimization under overhang constraints imposed by additive manufacturing technologies*, submitted (2017). HAL preprint: hal-01523009 (May 2017).

Deterministic submanifolds and analytic solution of the quantum stochastic differential master equation describing a monitored qubit

Pierre Rouchon
Centre Automatique et Systèmes, Mines-ParisTech, PSL Research University.
QUANTIC , INRIA Paris
pierre.rouchon@mines-paristech.fr

Joint work with A. Sarlette, Ph. Campagne-Ibarcq, P. Six,
L. Bretheau, M. Mirrahimi, and B. Huard

This talk is based on an experimental paper [1] and on a theoretical one [2]. It addressed the stochastic differential equation (SDE) associated to a two-level quantum system (qubit) subject to Hamiltonian evolution as well as unmonitored and monitored decoherence channels. The latter imply a stochastic evolution of the quantum state (density operator), whose associated probability distribution we characterize. We first show that for two sets of typical experimental settings, corresponding either to weak quantum non demolition measurements or to weak fluorescence measurements, the three Bloch coordinates of the qubit remain confined to a deterministically evolving surface or curve inside the Bloch sphere. We explicitly solve the deterministic evolution, and we provide a closed-form expres- sion for the probability distribution on this surface or curve. Then we relate the existence in general of such deterministically evolving submanifolds to an accessibility question of control theory, which can be answered with an explicit algebraic criterion on the SDE. This allows us to show that, for a qubit, the above two sets of weak measurements are essentially the only ones featuring deterministic surfaces or curves.

References

[1] P. Campagne-Ibarcq, P. Six, L. Bretheau, A. Sarlette, M. Mirrahimi, P. Rouchon, B. Huard: *Observing quantum state diffusion by heterodyne detection of fluorescence.* Phys. Rev. X, American Physical Society, 2016, 6, 011002

[2] A. Sarlette, P. Rouchon: *Deterministic submanifolds and analytic solution of the quantum stochastic differential master equation describing a monitored qubit.* Journal of Mathematical Physics. 58(6):062106, 2017.

Sufficient second-order conditions for bang-bang control problems

Eduardo Casas
University of Cantabria
eduardo.casas@unican.es

Joint work with D. Wachsmuth and G. Wachsmuth.

We provide sufficient optimality conditions for optimal control problems with bang-bang controls. Building on a structural assumption on the adjoint state, we additionally need a weak second-order condition. This second-order condition is formulated with functions from an extended critical cone, and it is equivalent to a formulation posed on measures supported on the set where the adjoint state vanishes. If our sufficient optimality condition is satisfied, we obtain a local quadratic growth condition in $L^1(\Omega)$.

Optimal control of systems of reaction-diffusion equations

Fredi Tröltzsch
Technische Universität Berlin, Berlin, Germany,
troeltzsch@math.tu-berlin.de

Joint work with E. Casas and C. Ryll.

The optimal control of a general system of nonlinear reaction-diffusion equations is considered including several important equations of mathematical physics. In particular, equations are covered that develop traveling wave fronts, spiral waves, scroll rings, or propagating spot solutions. Well-posedness of the system and differentiability of the control-to-state mapping are addressed. Associated optimal control problems with pointwise constraints on the control and the state are discussed. Moreover, necessary first-order optimality conditions are derived. Numerical examples illustrate a great diversity of moving patterns and their control. In particular a novel application of pointwise state constraints is presented.

Some applications of Hamiltonian systems in control theory

Dan Tiba
Institute of Mathematics (Romanian Academy)
dan.tiba@imar.ro

We discuss some Hamiltonian systems and prove basic properties, under the independence condition. We also investigate the critical case, define a class of generalized solutions and prove specific properties. Relevant examples and counterexamples are also indicated. The applications concern representations of implicitly defined manifolds and certain optimal control problems.

The turnpike property in optimal control

Emmanuel Trélat
Sorbonne Universités, UPMC Univ Paris 06, CNRS UMR 7598,
emmanuel.trelat@upmc.fr

Joint work with C. Zhang and E. Zuazua.

The turnpike property emerged in the 50's, after the works by the Nobel prize Samuelson in econometry. It stands for the general behavior of an optimal trajectory solution of an optimal control problem in large time. This trajectory trends to behave as the concatenation of three pieces: the first and the last arc being rapid transition arcs, and the middle one being in large time, almost stationary, close to the optimal value of an associated static optimal control problem.

In [1] we have established the turnpike property in a general framework in finite dimensional nonlinear optimal control. We prove that not only the optimal trajectory is exponentially close so some (optimal) stationary state, but also the control and the adjoint vector coming from the Pontryagin maximum principle. Our analysis shows an hyperbolicity phenomenon which is intrinsic to the symplectic feature of the extremal equations. We infer a very simple and efficient numerical method to compute optimal trajectories in that framework, with an appropriate variant of the shooting method. We have extended our analysis to more general contexts, involving PDEs, in [2, 3, 4].

References

[1] E. Trélat, E. Zuazua, *The turnpike property in finite-dimensional nonlinear optimal control*, J. Differential Equations **258** (2015), no. 1, 81–114.

[2] M. Gugat, E. Trélat, E. Zuazua, *Optimal Neumann control for the 1D wave equation: finite horizon, infinite horizon, boundary tracking terms and the turnpike property*, Syst. Cont. Letters **90** (2016), 61–70.

[3] E. Trélat, C. Zhang, E. Zuazua, *Steady-state and periodic exponential turnpike property for optimal control problems in Hilbert spaces*, Preprint Hal (2016), 26 pages.

[4] E. Trélat, C. Zhang, *Integral and measure-turnpike property for infinite-dimensional optimal control problems*, Preprint Hal (2017), 28 pages.

Thursday, 16 November

Optimal Control of Time Delay Systems

Richard Vinter
Imperial College, Exhibition Road, London SW7 2AZ, UK
r.vinter@imperial.ac.uk

This talk concerns optimal control of time delay systems. Time delays arise in many areas of control engineering, including process control where they are associated with transport delays of reagents flowing between reactors, and control applications in planetary exploration where communications delays are an important factor. Time delays are also encountered in economic modelling. The derivation of necessary conditions of optimality for time delay problems, in the form of a generalized Maximum Principle, date back to the early days of optimal control theory. But a number of important issues have until recently remained unresolved; these concern, in particular, optimality conditions covering free end-time problems, optimality conditions for problems involving non-commensurate delays in the control, versions of the necessary conditions which are valid for non-smooth data, refined optimality conditions valid in the autonomous case ('constancy of a generalized Hamiltonian' along an optimal trajectory) and conditions for existence of minimizers. We give an overview of the area and present some recent advances.

Necessary optimality conditions for a weak local minimum in general optimal control problem with Volterra-type integral equations on a variable time interval

Nikolai P. Osmolovskii
University of Technology and Humanities in Radom, Poland
osmolovski@uph.edu.pl

Joint work with A. V. Dmitruk.

We study an optimal control problem with a nonlinear Volterra-type integral equation considered on a nonfixed time interval, subject to endpoint constraints of equality and inequality type, mixed state-control constraints of inequality and equality type, and pure state constraints of inequality type. The main assumption is the linear–positive independence of the gradients of active mixed constraints with respect to the control. We obtain first-order necessary optimality conditions for an extended weak minimum, the notion of which is a natural generalization of the notion of weak minimum with account of variations of the time. The conditions obtained generalize the corresponding ones for problems with ordinary differential equations.

References

[1] J.F. Bonnans and C. De La Vega, *Optimal control of state constrained integral equations*, Set-Valued Var. Anal., v. 18 (3-4) (2010), 307–326.

[2] J.F. Bonnans, X. Dupuis, and C. De La Vega, *First and second order optimality conditions for optimal control problems of state constrained integral equations*, J. Optim. Theory Applic., v. 159(1) (2013), 1–40.

[3] A. V. Dmitruk and N. P. Osmolovskii, *Necessary conditions for a weak minimum in optimal control problems with integral equations subject to state and mixed constraints*, SIAM J. on Control and Optimization, v. 52, No. 6 (2014), 3437–3462.

[4] A.V. Dmitruk and N.P. Osmolovskii, *Necessary conditions for a weak minimum in optimal control problems with integral equations on a variable time interval*, Discrete and Continuous Dynamical Systems, Ser. A, v. 35, No. 9 (2015), 4323–4343.

Second order conditions for control-affine problems: a revised approach and extensions

Andrei Dmitruk
Russian Academy of Sciences, CEMI
vraimax@mail.ru

We will present a new approach to obtaining second (quadratic) order necessary and sufficient conditions for optimal control problems governed by systems linear (affine) in the control. For the weak minimum, this approach seems to be more natural and simple than those used in previous works. It also allows some extensions to other types of minimum and systems.

Controllability under constraints

Enrique Zuazua
DeustoTech, University of Deusto,
Universidad Autónoma de Madrid,
LJLL-Paris
enrique.zuazua@uam.es

Joint work with J. Lohéac, E. Trélat and D. Pighin.

The free heat equation is well known to preserve the non-negativity of solutions. On the other hand, due to the infinite velocity of propagation, the heat equation is null-controllable in an arbitrary small time interval. The following question then arises naturally: Can the heat dynamics be controlled under a positivity constraint on the state, requiring that the state remains non-negative all along the time dependent trajectory? We will show that, if the control time is large enough, constrained controllability holds. We will also show that it fails to be true if the control time is too short. In other words, despite of the infinite velocity of propagation, under the natural positivity constraint on the state, controllability fails when the time horizon is too short. Links with other related topics such as finite-dimensional systems, sparse control and the turnpike property will also be discussed.

Boundary stabilization of time periodic parabolic systems

Jean-Pierre Raymond
Université Paul Sabatier Toulouse III
Institut de Mathématiques de Toulouse
raymond@math.univ-toulouse.fr

Joint work with M. Badra, M. Ramaswamy and D. Mitra.

We are interested in the stabilization of fluid flows or fluid-structure systems around time periodic solutions. This work is motivated by understanding what is called 'autoregulation' of blood flow pressure in the brain. Our aim is to extend to time periodic systems results recently established for fluid flows [1, 7] or fluid structure systems [5, 6]. Since we deal with boundary controls, we have first extended results from [4] to the case of unbounded control operators.

Some related fluid-structure systems are currently studied by J.-J. Casanova (IMT, Toulouse & Monash University, Melbourne).

References

[1] C. Airiau, J.-M. Buchot, R. K. Dubey, M. Fournié, J.-P. Raymond, J. Weller-Calvo, Stabilization and best actuator location for the Navier-Stokes equations, to appear in SIAM J. Sc. Computing.

[2] M. Badra, D. Mitra, M. Ramaswamy, Local stabilization of time-periodic evolution equations, preprint 2017.

[3] A. Lunardi, Stabilizability of time-periodic parabolic equations, SIAM J. Control Optim. 29 (1987), 281-314.

[4] A. Lunardi, Bounded solutions of linear periodic abstract parabolic equations, Proc. Royal Soc. Edinburgh, A, 110 (1988), 135-159.

[5] M. Fournié, M. Ndiaye, J.-P. Raymond, Feedback stabilization of a two-dimensional fluid-structure interaction system with mixed boundary conditions, preprint 2017.

[6] M. Fournié, M. Ndiaye, J.-P. Raymond, Feedback stabilization of a fluid-structure interaction system in a wind tunnel, preprint 2017.

[7] P. A. Nguyen, J.-P. Raymond, Boundary stabilization of the Navier-Stokes equations in the case of mixed boundary conditions. SIAM J. Control Optim. 53 (2015), no. 5, 3006-3039.

A selection of applications of the Bocop toolbox

Pierre Martinon
Inria Saclay and CMAP Ecole Polytechnique
pierre.martinon@inria.fr

We present a selection of applications studied with the optimal control toolbox Bocop [1]. Bocop primarily implements a direct transcription method, and also provides a dynamic programming method. First example is climb and cruise trajectory optimization for civil airplanes, a work in collaboration with startup Safety Line. Then we illustrate a combined approach with indirect methods on contrast optimization for magnetic resonance imaging (MRI). Third example is the energy management of a microgrid, comparing dynamic programming for switched systems with the existing MILP approach. Finally we present a case of delay problem linked to the control of leukemic cell populations.

References

[1] Bonnans, F., Martinon, P., Giorgi, D., Grelard, V., Heymann, B., Jinyan, L., Maindrault, S., Tissot, O.: *Bocop - a collection of examples.* Tech. rep. (2017). **http://bocop.org/**

The New Butterfly Relaxation Methods for Mathematical Programs with Complementarity Constraint

Mounir Haddou
IRMAR-INSA Rennes,
mounir.haddou@insa-rennes.fr

Joint work with J.-P. Dussault and T. Migot.

We consider the Mathematical Program with Complementarity Constraints

$$\begin{array}{ll} \min\limits_{x\in\mathbb{R}^n} & f(x) \\ \text{s.t.} & g(x) \leq 0, h(x) = 0, \\ & 0 \leq G(x) \perp H(x) \geq 0, \end{array} \tag{1}$$

with $f : \mathbb{R}^n \to \mathbb{R}$, $h : \mathbb{R}^n \to \mathbb{R}^m$, $g : \mathbb{R}^n \to \mathbb{R}^q$ and $G, H : \mathbb{R}^n \to \mathbb{R}^c$ that are assumed continuously differentiable. The notation $0 \leq u \perp v \geq 0$ for two vectors u and v in $\mathbb{R}^c$ is a shortcut for $u_i \geq 0,\ v_i \geq 0$ and $u_i v_i = 0$ for all $i \in \{1, \dots, c\}$.

We propose a new family of relaxation schemes for mathematical programs with complementarity constraints that extends the relaxations converging to an M-stationary point [1, 2, 3]. We discuss the properties of the sequence of relaxed non-linear programs as well as stationarity properties of limiting points. We prove under a new and weak constraint qualification, that our relaxation schemes have the desired property of converging to an M-stationary point.

Unfortunately, in practice, relaxed problems are only solved up to approximate stationary points and the guarantee of convergence to an M-stationary point is lost (c.f [4]).

We define a new strong approximate stationarity condition and prove that we can maintain our guarantee of convergence and attain the desired goal of computing an M-stationary point.

A comprehensive numerical comparison between existing relaxations methods is performed and shows promising results for our new methods.

We also propose different extensions to tackle MPVC (vanishing constraints) and MOCC (cardinality constraints) problems.

References

[1] Flegel, Michael L and Kanzow, Christian, *Abadie-type constraint qualification for mathematical programs with equilibrium constraints* , Journal of Optimization Theory and Applications, 2005, V.124 (3), pp 595–614 .

[2] Kadrani, Abdeslam and Dussault, Jean-Pierre and Benchakroun, Abdelhamid, *A new regularization scheme for mathematical programs with complementarity constraints*, SIAM Journal on Optimization, 2009, V. 20 (1), pp 78–103.

[3] Schwartz, Alexandra, *Mathematical programs with complementarity constraints: Theory, methods, and applications*, Ph. D. dissertation, Institute of Applied Mathematics and Statistics, University of Würzburg, 2011.

[4] Kanzow, Christian and Schwartz, Alexandra, *The Price of Inexactness: Convergence Properties of Relaxation Methods for Mathematical Programs with Complementarity Constraints Revisited* , Mathematics of Operations Research, 2015, V.40(2), pp 253–275.

Viscosity Solutions of Systems of PDEs with Interconnected Obstacles and Switching Problem without Monotonicity Condition

Mohamed Mnif
University of Tunis El Manar, LAMSIN
mohamed.mnif@enit.rnu.tn

Joint work with S. Hamadène and S. Neffati.

We show existence and uniqueness of a continuous viscosity solution of a system of partial differential equations (PDEs for short) without assuming the usual monotoniciy condition on the driver function. Our method strongly relies on the link between PDEs and reflected backward stochastic differential equations with interconnected obstacles for which we already know that the solution exists and is unique for general drivers.

A steady-state and dynamical analysis for the optimization of the bio-gas production in a chemostat model

Térence Bayen
Université de Montpellier
terence.bayen@umontpellier.fr

Joint work with P. Gajardo.

We address the problem of finding an optimal feedback control maximizing the production of bio-gas over a given period in a continuous bioreactor described by the so-called AM2 model [2]. To do so, we first analyze the optimization problem at the steady state *i.e.* we maximize the static production of bio-gas among equilibria of the controlled system with constant dilution rates. Since the bioreactor can be stabilized at the corresponding equilibrium point, we then consider the minimum time problem to reach this equilibrium. The system can be then expressed as an affine system with a drift and one controlled vector field. It appears that the target point belongs to the collinearity set *i.e.* the set of points where the drift is parallel to the the controlled vector field. Thanks to Pontryagin's Principle and geometric control theory, we provide an optimal feedback control for this problem. This is a joint work with Pedro Gajardo from University of Santa Maria, Chile [1].

References

[1] T. Bayen, P. Gajardo, *A note on the steady state optimization of the biogas production in a two-stages anaerobic digestion model*, https://hal.archives-ouvertes.fr/hal-01586942 submitted, 2017.

[2] J. Harmand, C. Lobry, A. Rapaport, T. Sari, *Le chémostat: théorie mathématique de la culture continue de micro-organismes*, ISTE Editions, Collection Génie des Procédés, Série Chémostat et bioprocédés, Vol. 1., 2017.

Dynamic programming over noncommutative spaces

Stéphane Gaubert
INRIA and CMAP, École polytechnique, CNRS
Stephane.Gaubert@inria.fr

Joint work with N. Stott.

McEneaney's max-plus basis method allows one to approximate the value function of a deterministic optimal control problem by a supremum of elementary functions like quadratic forms. Recently, Ahmadi et al. developed an approximation method for Barabanov norms of switched linear systems, relying also on the approximation by suprema of quadratic forms. Related methods were developed in computer science, they allow one to compute program invariants, represented as intersections or unions or ellipsoids. In all these approaches, the solution of large scale linear matrix inequalities by semidefinite programming methods is the computational bottleneck. We will show that the recourse to semidefinite programming can be avoided by expressing piecewise quadratic invariant generation and value function approximation as a fixed point problem in the space of positive semidefinite matrices. To do so, we introduce "noncommutative" analogues of Bellman operators, acting on spaces of matrices, instead of spaces of functions. The minima and maxima of functions are now replaced by selections of minimal upper bounds and maximal upper bounds, with respect to the Loewner order. By a theorem of Kadison, such selections are not unique. They are also not order preserving. However, the Bellman type operators which arise in this way retain a remarkable structure. Indeed, they appear as the tropicalizations of the completely positive maps representing quantum channels, and the convergence of numerical schemes can be established by exploiting metric properties of the cone of positive definite matrices. This allows us to obtain relaxed solutions for very large scale instances (e.g., approximations of the joint spectra radius in dimension 500).

This is based on the recent work [5], and also on [2,3,4].

References

[1] A. A. Ahmadi, R. M. Jungers, P. A. Parrilo, and M. Roozbehani. Joint spectral radius and path-complete graph Lyapunov functions. *SIAM J. Control and Optimization*, 52(1):687–717, 2014.

[2] X. Allamigeon, S. Gaubert, N. Stott, É. Goubault, and S. Putot. A scalable algebraic method to infer quadratic invariants of switched systems. *ACM Trans. Embed. Comput. Syst.*, 15(4):69:1–69:20, September 2016.

[3] X. Allamigeon, S. Gaubert, N. Stott, É. Goubault, and S. Putot. A Fast Method to Compute Disjunctive Quadratic Invariants of Numerical Programs. *Proc. of EMSOFT 2017*, Seoul.

[4] S. Gaubert and Z. Qu. The contraction rate in Thompson's part metric of order-preserving flows on a cone - application to generalized Riccati equations. *Journal of Differential Equations*, 256(8):2902–2948, 2014.

[5] S. Gaubert and N. Stott. Tropical Kraus maps for optimal control of switched systems, preprint, to appear in the Proceedings of CDC'2017, Melbourne arXiv:1706.04471

[6] W. M. McEneaney. A curse-of-dimensionality-free numerical method for solution of certain HJB PDEs. *SIAM journal on Control and Optimization*, 46(4):1239–1276, 2007.

Friday, 17 November

Risk-Averse Control of Partially Observable Markov Systems

Andrzej Ruszczynski
Rutgers University
rusz@rutgers.edu

We consider risk measurement in controlled partially observable Markov systems in discrete time. In such systems, part of the state vector is not observed, but affects the transition kernel and the costs. We introduce new concepts of risk filters and study their properties. We also introduce the concept of conditional stochastic time consistency. We derive the structure of risk filters enjoying this property and prove that they can be represented by a collection of law invariant risk measures on the space of function of the observable part of the state. We also derive the corresponding dynamic programming equations. Then we illustrate the results on a clinical trial problem and a machine deterioration problem. In the final part of the talk, we shall discuss risk filtering and risk-averse control of partially observable Markov systems in continuous time.

Probabilistic Programming: Structural Properties and Applications

René Henrion
Weierstrass Institute Berlin
henrion@wias-berlin.de

An optimization problem subject to probabilistic (chance) constraints has the general form

$$\min\{f(x) \mid \mathbb{P}(g(x,\xi) \geq 0) \geq p\}$$

where x is a decision variable, f is an objective function, ξ is a random vector, g is a (random) constraint mapping, $\mathbb{P}$ refers to a probability measure and $p \in (0,1)$ is some probability level. The inequality above (called probabilistic constraint) defines a decision to be feasible if the random inequality system is satisfied with at least probability p. Applications of such problems are abundant in engineering, namely in power management. Traditionally, they are embedded into the area of operations research, i.e. with finite-dimensional decisions. Recently, there has been growing interest in probabilistic state constraints in PDE constrained optimization. This requires new investigations about continuity, differentiability, convexity of such problems in an infinite dimensional setting. The talk provides some recent results [1, 2] in this direction along with a few applications.

References

[1] A. Hantoute, R. Henrion and P. Pérez-Aros, *Subdifferential characterization of probability functions under Gaussian distribution*, submitted.

[2] M.H. Farshbaf-Shaker, R. Henrion and D. Hömberg, *Properties of chance constraints in infinite dimensions with an application to PDE constrained optimization*, to appear in: Set-Valued and Variational Analysis.

Properties of probability functions and their level sets.

Wim van Ackooij
EDF R&D, 7 Boulevard Gaspard Monge
wim.van-ackooij@edf.fr

Joint work with R. Henrion and J. Malick.

In this talk we will discuss some recent insights obtained in the study of probability functions and their level sets. A probability function results from a measure acting on a random inequality system depending both on a decision and random vector. Level sets of such probability functions translate the desire that such a system holds with high enough probability in order to ensure safety of the decision. We will highlight that under some additional assumptions on the nominal inequality system, certain level sets are actually convex. We will also briefly mention several recent insights obtained on the differentiability of probability functions.

References

[1] W. van Ackooij and R. Henrion. Gradient formulae for nonlinear probabilistic constraints with Gaussian and Gaussian-like distributions. *SIAM Journal on Optimization*, 24(4):1864–1889, 2014.

[2] W. van Ackooij and R. Henrion. (Sub-) Gradient formulae for probability functions of random inequality systems under Gaussian distribution. *SIAM Journal on Uncertainty Quantification*, 5(1):63–87, 2017.

[3] W. van Ackooij and J. Malick. Eventual convexity of probability constraints with elliptical distributions. *Submitted Preprint*, pages 1–20, 2017.

Optimal stopping games with nonlinear expectations

Agnès Sulem
INRIA Paris, Equipe-projet MathRisk
agnes.sulem@inria.fr

Joint work with R. Dumitrescu and M.-C. Quenez.

We present some recent results for optimal stopping and Dynkin stochastic games under nonlinear expectation and study pricing and hedging issues for game options in an imperfect financial market with default. Imperfections in the market are taken into account via the nonlinearity of the wealth dynamics. In this framework, the pricing system is expressed as a nonlinear $\mathscr{E}^g$-expectation/evaluation induced by a nonlinear Backward Stochastic Differential Equation (BSDE) with jump with driver g. We prove that the superhedging price of a game option coincides with the value function of a corresponding *generalized* Dynkin game expressed in terms of the $\mathscr{E}^g$-evaluation. Th proofs of these results are based on links between generalized Dunkin games and doubly reflected BSDEs. In the Markovian case, we provide a probabilistic interpretation of semi-linear PDEs with two barriers in terms of generalized game problems. We also address the case of ambiguity on the model, - for example an ambiguity on the default probability - and characterize the superhedging price of a game option as the value function of a *mixed generalized* Dynkin game.

References

[1] Dumitrescu, R., Quenez M.C., Sulem A., (2014) *Generalized Dynkin Games and Doubly reflected BSDEs with jumps*, Electronic Journal of Probability, (2016), 21, n°4, pp 1–32.

[2] Dumitrescu, R., Quenez M.C., Sulem A., (2015), *Mixed Generalized Dynkin Games and Stochastic control in a Markovian framework*, Stochastics (2016), pp 1-30

[3] Dumitrescu R., M.C. Quenez and A. Sulem, *Weak Dynamic Programming Principle for Combined Optimal Stopping/Stochastic Control with $\mathscr{E}^f-$conditional Expectations*, SIAM J. Control and Optimization, (2016) Vol. 54, No. 4, pp. 2090–2115.

[4] Dumitrescu R., M.C. Quenez and A. Sulem, *Optimal Stopping for Dynamic Risk Measures with Jumps and Obstacle Problems*, Journal Optimization Theory and Applications, Volume 167, Issue 1 (2015), 219–242.

[5] El Karoui N. and M.C. Quenez, *Non-linear Pricing Theory and Backward Stochastic Differential Equations*, Lectures Notes in Mathematics 1656, Bressanone, 1996, Editor: W.J.Runggaldier, collection Springer,1997.

[6] Pardoux E. and S. Peng, *Adapted solution of a backward stochastic differential equation*, Systems Control Letters, 14(1), 55-61 (1990),

[7] Peng, S. Nonlinear expectations, nonlinear evaluations and risk measures, 165-253, *Lecture Notes in Math.*, Springer, Berlin 2004.

[8] Quenez M.-C. and Sulem A., *BSDEs with jumps, optimization and applications to dynamic risk measures,* Stochastic Processes and Applications 123 (2013) 3328-3357.

[9] Quenez M.-C. and Sulem A., *Reflected BSDEs and robust optimal stopping for dynamic risk measures with jumps,* Stochastic Processes and Applications 124, (2014) 3031–3054.

Nonlinear representation and application to stochastic differential games

Nizar Touzi
CMAP, Ecole Polytechnique
nizar.touzi@polytechnique.edu

In a Brownian filtration every centered random variable can be written as a stochastic integral with respect to the underlying Brownian motion. This result corresponds to the path-dependent version of the heat equation. We prove analogous representation results corresponding to path-dependent versions of fully nonlinear second order parabolic PDEs. As an application, we show how to reduce a classical zero-sum stochastic differential game, known as the *principal agent problem* in Economics, to a standard stochastic optimal control problem.